Geometric phase for an accelerated two-level atom and the Unruh effect

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Abstract

We study, in the framework of open quantum systems, the geometric phase acquired by a uniformly accelerated two-level atom undergoing nonunitary evolution due to its coupling to a bath of fluctuating vacuum electromagnetic fields in the multipolar scheme. We find that the phase variation due to the acceleration can be in principle observed via atomic interferometry between the accelerated atom and the inertial one, thus providing an evidence of the Unruh effect.

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When a quantum system undergoes a cyclic evolution, it may acquire a memory of this motion in the form of a geometric phase. This phase was first introduced by Pancharatnam while studying polarized beams passing through crystals [1]. In 1984, Berry studied the dynamics of a closed quantum system whose Hamiltonian varies adiabatically in a cyclic way, and found, besides the familiar dynamical phase, that there is an additional phase due to the geometry of the path enclosed during the evolution of the system in the parameter space [2]. Berry's work was soon generalized to nonadiabatic [3] and noncyclic evolution [4]. The geometric phase has so far been extensively studied, both theoretically and experimentally [5], and it has been fruitfully applied to many fields, such as the study of molecular dynamics [6] and electronic properties [7].

Recently, there has been interest in using the geometric phase for fault-tolerant quantum computation [8]. However, due to the inevitable interactions between the qubits and the environment, a pure state will be driven to a mixed state under the environment induced decoherence and dissipation. As a result, the geometric phase has to be generalized to general evolutions of an open system. Uhlmann was the first to define a mixed-state geometric phase via mathematical concept of purification [9]. Sjöqvist et al. put forward an alternative definition for the unitarily evolved nondegenerate mixed-state density matrix based on the interferometry [10]. This was soon generalized to degenerate mixed states by Singh et al. [11] and to the nonunitary evolution using the kinematic approach by Tong et al. [12]. Wang et al. defined a mixed-state geometric phase via mapping the density matrix to a nonunit vector ray in the complex projective Hilbert space [13]. Experiments based on NMR system [14] and single photon interferometry [15] have demonstrated the mixed-state geometric phase.

As discussed above, the impact of environment on the geometric phase of open systems is an important issue in any practical implementations of quantum computing. In this regard, the effects of different kinds of decoherence sources on the geometric phase, such as dephasing and spontaneous decay, haven been analyzed [16]. In Ref [17], Rezakhani et al. have studied geometric phase for an open system, which is a spin-half particle in weak coupling to a thermal bath, and found that the phase varies with the temperature of the bath. Lombardo et al. [18] have studied not only how the geometric phase is modified by the presence of

the different types of environments, but also estimated the corresponding times at which decoherence becomes effective. Chen et al. [19] focused on the geometric phase of an open two-level atom coupled to an environment with Lorentzian spectral density and explored the non-Markovian effect on the geometric phase.

In quantum sense, every system, whatever it is, is an open system, since it is at least subjected to vacuum fluctuations. However, the geometric phase of an open system generated by the nonunitary evolution due to its coupling to vacuum fluctuations is in general unobservable, as, practically, any phase variation is observed only via some kind of interferometry between the involved state and certain selected reference states which are both inseparably coupled to vacuum. Nevertheless, if, somehow, vacuum fluctuations are modified, then the geometric phase of the nonunitary evolution of an open system caused by its coupling to vacuum may become potentially observable. The modification of vacuum fluctuations induced by the acceleration of a two-level atom, for example, may provide such as a possibility, since, as is well-known, a uniformly accelerated observer perceives the Minkowski vacuum as a thermal bath of Rindler particles [20]. This is the so-called Unruh effect. So, the phase variation due to the acceleration of an two-level atom, which can in principle be observed through interference with an inertial atom, may provide evidence of the Unruh effect which is deeply related to the Hawking radiation. In this regard, let us note that many novel proposals have been suggested to detect the Unruh effect and the Hawking radiation in analog systems [21]. At this point, it may be worth pointing out that the Unruh effect is associated with quantization of the field in the Rindler accelerated frame. However, theoretical calculations performed from the perspectives of both the inertial frame and the Rindler accelerated frame with the Unruh thermal bath usually produce the same result on physical observables [22], as is the case in the weak decay of a uniformly accelerated proton [23], the bremsstrahlung effect associated with a uniformly accelerated point charge [24, 25], and the spontaneous excitation of a uniformly accelerated atom [26]

Recently, Martin-Martinez et al. [27] have considered the possibility of using geometric phase to detect the Unruh effect. They examined an accelerated detector modeled by a harmonic oscillator which couples only to a single-mode of a scalar field in vacuum, and

calculated the geometric phase acquired by the joint state of the detector and the field. As a result, cavities which are leaky to a finite number of modes are essential for the measurement of the acceleration influence in order to realize the single mode coupling and avoid the problems in the Unruh effect itself arising from introduction of boundaries. Such kind of cavities seems to be a major challenge in experimental implementation of their proposal. Here, we would like to consider a more realistic case and propose using the geometric phase of non-unitary evolution to detect the Unruh effect. We plan to study an accelerated two-level system which couples to all vacuum modes of electromagnetic (rather than scalar) fields in a realistic multipolar coupling scheme [28]. We treat the accelerated two-level atom as an open system [1] in a reservoir of fluctuating vacuum electromagnetic fields and calculate the geometric phase of the accelerated open system undergoing non-unitary evolution because of the environment induced decoherence and dissipation. Since in our study, the atom couples to all vacuum modes, no cavity is needed in any experimental scheme to detect the phase. At this point, it is worth noting that the quantum geometric phase of an open system undergoing nonunitary evolution due to its coupling to a quantum critical bath has recently been demonstrated using a NMR quantum simulator [32].

Let us write the total Hamiltonian of the system (atom plus reservoir) as $H = H_s + H_\phi + H'$. Here H_s is the Hamiltonian of the atom, and, for simplicity, is taken to be $H_s = \frac{1}{2}\,\hbar\omega_0\sigma_3$, in which σ_3 is the Pauli matrix. ω_0 is the energy level spacing of the atom. H_ϕ is the Hamiltonian of the free electromagnetic field, of which the details are not needed here. The Hamiltonian that describes the interaction between the atom and the electromagnetic field in the multipolar coupling scheme is given by $H'(\tau) = -e\mathbf{r}\cdot\mathbf{E}(x(\tau)) = -e\sum_{mn}\mathbf{r}_{mn}\cdot\mathbf{E}(x(\tau))\sigma_{mn}$, where e is the electron electric charge, $e\,\mathbf{r}$ the atomic electric dipole moment, and $\mathbf{E}(x)$ the electric field strength.

At the beginning, the whole system is characterized by the total density matrix $\rho_{tot} = \rho(0)\otimes|0\rangle\langle 0|$, in which $\rho(0)$ is the initial reduced density matrix of the atom, and $|0\rangle$ is the

[1] Let us note that the theory of open quantum system has been fruitfully applied to understand, from a different perspective, the Unruh, Hawking and Gibbons-Hawking effects, in Ref. [29], [30] and [31], respectively.

vacuum state of the field. In the frame of the atom, the evolution in the proper time τ of the total density matrix ρ_{tot} satisfies

$$\frac{\partial \rho_{tot}(\tau)}{\partial \tau} = -\frac{i}{\hbar}[H, \rho_{tot}(\tau)] \ . \tag{1}$$

We assume that the interaction between the atom and the field is weak. In the limit of weak coupling, the evolution of the reduced density matrix $\rho(\tau)$ can be written in the Kossakowski-Lindblad form [33, 34]

$$\frac{\partial \rho(\tau)}{\partial \tau} = -\frac{i}{\hbar}\big[H_{\rm eff},\, \rho(\tau)\big] + \mathcal{L}[\rho(\tau)] \ , \tag{2}$$

where

$$\mathcal{L}[\rho] = \frac{1}{2}\sum_{i,j=1}^{3} a_{ij}\big[2\,\sigma_j \rho\,\sigma_i - \sigma_i\sigma_j\,\rho - \rho\,\sigma_i\sigma_j\big] \ . \tag{3}$$

The matrix a_{ij} and the effective Hamiltonian $H_{\rm eff}$ are determined by the Fourier and Hilbert transforms of the field correlation functions

$$G^{+}(x-y) = \frac{e^2}{\hbar^2}\sum_{i,j=1}^{3}\langle +|r_i|-\rangle\langle -|r_j|+\rangle\,\langle 0|E_i(x)E_j(y)|0\rangle \ , \tag{4}$$

which are defined as follows

$$\mathcal{G}(\lambda) = \int_{-\infty}^{\infty} d\tau\, e^{i\lambda\tau}\, G^{+}\big(x(\tau)\big) \ , \qquad \mathcal{K}(\lambda) = \frac{P}{\pi i}\int_{-\infty}^{\infty} d\omega\, \frac{\mathcal{G}(\omega)}{\omega-\lambda} \ . \tag{5}$$

Then the coefficients of the Kossakowski matrix a_{ij} can be written as

$$a_{ij} = A\delta_{ij} - iB\epsilon_{ijk}\delta_{k3} + C\delta_{i3}\delta_{j3} \ , \tag{6}$$

in which

$$A = \frac{1}{4}[\mathcal{G}(\omega_0) + \mathcal{G}(-\omega_0)] \ , \quad B = \frac{1}{4}[\mathcal{G}(\omega_0) - \mathcal{G}(-\omega_0)] \ , \quad C = -A \ . \tag{7}$$

The effective Hamiltonian $H_{\rm eff}$ contains a correction term, the so-called Lamb shift, and one can show that it can be obtained by replacing ω_0 in H_s with a renormalized energy level spacing Ω as follows

$$H_{\rm eff} = \frac{1}{2}\hbar\Omega\sigma_3 = \frac{\hbar}{2}\{\omega_0 + \frac{i}{2}[\mathcal{K}(-\omega_0) - \mathcal{K}(\omega_0)]\}\,\sigma_3 \ . \tag{8}$$

For convenience, let us express the density matrix ρ in terms of the Pauli matrices,

$$\rho(\tau) = \frac{1}{2}\left(1 + \sum_{i=1}^{3} \rho_i(\tau)\sigma_i\right) . \tag{9}$$

Plugging Eq. (9) into Eq. (2) and assuming that the initial state of the atom is $|\psi(0)\rangle = \cos\frac{\theta}{2}|+\rangle + \sin\frac{\theta}{2}|-\rangle$, we can easily work out the time-dependent reduced density matrix

$$\rho(\tau) = \begin{pmatrix} e^{-4A\tau}\cos^2\frac{\theta}{2} + \frac{B-A}{2A}(e^{-4A\tau}-1) & \frac{1}{2}e^{-2(2A+C)\tau - i\Omega\tau}\sin\theta \\ \frac{1}{2}e^{-2(2A+C)\tau + i\Omega\tau}\sin\theta & 1 - e^{-4A\tau}\cos^2\frac{\theta}{2} - \frac{B-A}{2A}(e^{-4A\tau}-1) \end{pmatrix} . \tag{10}$$

The geometric phase for a mixed state undergoing nonunitary evolution is given by [12]

$$\gamma = \arg\left(\sum_{k=1}^{N} \sqrt{\lambda_k(0)\lambda_k(T)}\langle\phi_k(0)|\phi_k(T)\rangle e^{-\int_0^T \langle\phi_k(\tau)|\dot{\phi}_k(\tau)\rangle d\tau}\right) , \tag{11}$$

where $\lambda_k(\tau)$ and $|\phi_k(\tau)\rangle$ are the eigenvalues and eigenvectors of the reduced density matrix $\rho(\tau)$. In order to get the geometric phase, we first calculate the eigenvalues of the density matrix (10) to get

$$\lambda_{\pm}(\tau) = \frac{1}{2}(1 \pm \eta) , \tag{12}$$

where $\eta = \sqrt{\rho_3^2 + e^{-4(2A+C)\tau}\sin^2\theta}$ and $\rho_3 = e^{-4A\tau}\cos\theta + \frac{B}{A}(e^{-4A\tau}-1)$. It is obvious that $\lambda_-(0) = 0$. As a result, the contribution comes only from the eigenvector corresponding to λ_+

$$|\phi_+(\tau)\rangle = \sin\frac{\theta_\tau}{2}|+\rangle + \cos\frac{\theta_\tau}{2}e^{i\Omega\tau}|-\rangle , \tag{13}$$

where

$$\tan\frac{\theta_\tau}{2} = \sqrt{\frac{\eta+\rho_3}{\eta-\rho_3}} . \tag{14}$$

The geometric phase can be calculated directly using Eq. (11)

$$\gamma = -\Omega\int_0^T \cos^2\frac{\theta_\tau}{2}\, d\tau . \tag{15}$$

Let us now calculate the geometric phase of an two-level atom which is uniformly accelerated, for example, in the x-direction. The trajectory of the atom is then described by

$$t(\tau) = \frac{c}{a}\sinh\frac{a\tau}{c}, \quad x(\tau) = \frac{c^2}{a}\cosh\frac{a\tau}{c}, \quad y(\tau) = z(\tau) = 0 . \tag{16}$$

In order to get the explicit form of the geometric phase, we need the field correlation functions, which can be worked out using the two point function of the electric field

$$\langle E_i(x(\tau))E_j(x(\tau'))\rangle = \frac{\hbar c}{4\pi^2\varepsilon_0}(\partial_0\partial_0'\delta_{ij} - \partial_i\partial_j')\frac{1}{|\mathbf{x}-\mathbf{x}'|^2-(c\,t-c\,t'-i\varepsilon)^2}\ . \tag{17}$$

The field correlation function for the trajectory (16) can then be evaluated from (17) in the frame of the atom to get

$$G^+(x,x') = \frac{e^2|\langle -|\mathbf{r}|+\rangle|^2}{16\pi^2\varepsilon_0\hbar c^7}\frac{a^4}{\sinh^4[\frac{a}{2c}(\tau-\tau'-i\varepsilon)]}\ . \tag{18}$$

So, the Fourier transform of the field correlation function is

$$\mathcal{G}(\lambda) = \frac{\lambda^3\,e^2|\langle -|\mathbf{r}|+\rangle|^2}{6\pi\varepsilon_0\hbar c^3}\left(1+\frac{a^2}{c^2\lambda^2}\right)\left(1+\coth\frac{\pi c\lambda}{a}\right)\ . \tag{19}$$

Consequently, the coefficients of the Kossakowski matrix a_{ij} and the effective level spacing of the atom are given by

$$A_a = -C_a = \frac{1}{4}\gamma_0\left(1+\frac{a^2}{c^2\omega_0^2}\right)\frac{e^{2\pi c\,\omega_0/a}+1}{e^{2\pi c\,\omega_0/a}-1}\ ,\qquad B_a = \frac{1}{4}\gamma_0\left(1+\frac{a^2}{c^2\omega_0^2}\right)\ , \tag{20}$$

$$\Omega_a = \omega_0 + \frac{\gamma_0 P}{2\pi\omega_0^3}\int_0^\infty d\omega\,\omega^3\left(\frac{1}{\omega+\omega_0}-\frac{1}{\omega-\omega_0}\right)\left(1+\frac{a^2}{c^2\omega^2}\right)\left(1+\frac{2}{e^{2\pi c\,\omega/a}-1}\right)\ , \tag{21}$$

where $\gamma_0 = e^2|\langle -|\mathbf{r}|+\rangle|^2\,\omega_0^3/3\pi\varepsilon_0\hbar c^3$ is the spontaneous emission rate. Then the geometric phase can be obtained according to

$$\gamma_a = -\int_0^T \frac{1}{2}\left(1-\frac{R-R\,e^{4A_a\tau}+\cos\theta}{\sqrt{e^{4A_a\tau}\sin^2\theta+(R-R\,e^{4A_a\tau}+\cos\theta)^2}}\right)\Omega_a\,d\tau\ , \tag{22}$$

where $R = B_a/A_a$. So, the phase accumulates as the system evolves, although the accumulation with time is not linear as in the unitary evolution case. For a single period of evolution, the result of this integral can be expressed as

$$\gamma_a = \frac{\Omega_a}{\omega_0}\big[F(2\pi)-F(0)\big]\ , \tag{23}$$

where the function $F(\varphi)$ is defined as

$$\begin{aligned} F(\varphi) \ = \ & -\frac{1}{2}\varphi - \frac{1}{8A_a}\ln\left(\frac{1-Q^2-R^2+2R^2\,e^{4A_a\varphi/\omega_0}}{2R}+S(\varphi)\right) \\ & -\frac{1}{8A_a}\mathrm{sgn}(Q)\ln\left(1-Q^2-R^2+2Q^2e^{-4A_a\varphi/\omega_0}+2\,|Q|S(\varphi)\,e^{-4A_a\varphi/\omega_0}\right)\ , \end{aligned} \tag{24}$$

in which $S(\varphi)=\sqrt{R^2\,e^{8A_a\varphi/\omega_0}+(1-Q^2-R^2)e^{4A_a\varphi/\omega_0}+Q^2}$, $Q=R+\cos\theta$ and $\mathrm{sgn}(Q)$ is the standard sign function. For small γ_0/ω_0, which is generally true as we will see later, we can perform a series expansion to the result. For a single quasi-cycle, we find, to the first order [2],

$$\gamma_a\approx-\pi(1-\cos\theta)-\pi^2\frac{\gamma_0}{2\omega_0}\sin^2\theta\left(1+\frac{a^2}{c^2\omega_0^2}\right)\left(2+\cos\theta+\frac{2}{e^{2\pi c\,\omega_0/a}-1}\cos\theta\right)\,. \tag{25}$$

The first term $-\pi(1-\cos\theta)$ in the above equation is the geometric phase we would have obtained if the system were isolated from the environment, and the second term is the correction induced by the interaction between the accelerated atom and the environment. The geometric phase contains a term proportional to a^2 apart from the usual thermal term with a Planckian factor, and this term becomes appreciable when the acceleration is of the order of $c\,\omega_0$, thus it enhances the accumulation of the geometric phase in contrast with the scalar field case where this term is absent. Let us note here that similar a^2 terms also appear in the studies of the energy shift [35] and the spontaneous excitation [36] of an accelerated atom once the scalar filed is replaced by the electromagnetic field. In the limit of $a\to 0$, which corresponds to the case of an inertial atom, there is still a correction, which comes from the zero point fluctuations of the Minkowski vacuum. The explicit form of this term reads

$$\gamma_I\approx-\pi(1-\cos\theta)-\pi^2\frac{\gamma_0}{2\omega_0}(2+\cos\theta)\sin^2\theta\,. \tag{26}$$

This correction is exactly the same as the one in Ref. [19], which is obtained by assuming an environment with a Lorentzian spectral density, and is very similar to the result in Ref. [37] derived from a different model. Thus the correction to the geometric phase purely due to the acceleration is

$$\delta_a=\gamma_a-\gamma_I\approx-\pi^2\frac{\gamma_0}{2\omega_0}\left[\frac{a^2}{c^2\omega_0^2}(2+\cos\theta)+\left(1+\frac{a^2}{c^2\omega_0^2}\right)\frac{2}{e^{2\pi c\,\omega_0/a}-1}\cos\theta\right]\sin^2\theta\,. \tag{27}$$

This reveals that the geometric phase difference between the accelerated and inertial atoms depends on the properties of the atom (transition frequency ω_0 and the spontaneous emission

[2] Here we have omitted the Lamb shift terms, since it is obvious that these terms contain a factor γ_0/ω_0 and they will only contribute to the phase at the second and higher orders of γ_0/ω_0.

rate γ_0), the initial state (angle θ), and the acceleration a. If we assume that $|\langle -|\mathbf{r}|+\rangle|$ is of the order of the Bohr radius a_0, and ω_0 of the order of $E_0/\hbar$, where $E_0 = -e^2/8\pi\varepsilon_0 a_0$ is the energy of the ground-state, then γ_0/ω_0 is of the order of 10^{-6}. For a given initial state, the phase difference increases with the acceleration, and it becomes significant when the acceleration is of the order $c\,\omega_0$. The initial state of the atom, i.e., the initial angle θ in the Bloch sphere representation, also plays an important role. When $\theta = 0$ and $\theta = \pi$, which corresponds to an initial excited state and an ground state respectively, the phase difference vanishes, whereas it reaches its maximum in the regime near $\theta = \pi/2$. For a typical transition frequency of the hydrogen atom, i.e., $\omega_0 \sim 10^{15}\ \mathrm{s}^{-1}$, the acceleration needed to observe this effect is of the order of $10^{23}\ \mathrm{m/s}^2$, which is extremely high. However, if we consider two-level systems with lower frequency, the acceleration needed can be smaller. If we choose transition frequencies of the atom in the microwave regime, for example, $\omega_0 = 2.0 \times 10^9\ \mathrm{s}^{-1}$, which is physically accessible [38, 39], then, for $a = 4\,c\,\omega_0 = 2.4 \times 10^{18}\ \mathrm{m/s}^2$, the phase difference can reach 1.6×10^{-4} rad after a single period of evolution, which may be within the current experimental precision.

The geometric phase discussed above, therefore, may be detected with an atom interferometer. One first prepares the two-level atom in a superposition of upper and lower states in a Ramsey zone. In one arm of the interferometer the atoms move inertially, and in the other arm the atoms are accelerated. An interferometric measurement is taken when the atoms in the two arms meet. Here, let us recall that our calculations of the geometric phase are based on the comoving frame of reference of the atom. So, in the example we consider, according to Eq. (16), a single period for the accelerated atom in the comoving frame $T = 2\pi/\omega_0 \sim 3.1 \times 10^{-9}$ s would transfer to a time interval of 5.1 s in the laboratory frame. Thus, one should prepare an inertial atom which moves fast enough so that a single period of time in its own frame also transfers to the same amount of time in the laboratory frame when the interference experiment is performed. A tricky point is whether the field to accelerate the atoms will change the structure of a real atom or even ionize it.

Another delicate issue in experimental implementation is how to cancel the dynamical phase that the atoms acquire. For systems under nonunitary evolution like what we are con-

sidering here, the removal of the dynamical phase from the total phase is a subtle issue [40]. However, since our purpose is to detect the Unruh effect associated with the acceleration of the atom, we do not really need a complete cancellation of the dynamical phase. Instead, we may choose slightly different paths to control the relative dynamical phase to be much smaller than the geometric phase acquired in one period, so the result is effectively dominated by the geometric phase difference between the accelerated and inertial atoms.

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[1] S. Pancharatnam, Proc. Indian Acad. Sci. A **44**, 247 (1956).

[2] M. V. Berry, Proc. R. Soc. Lond. A **392**, 45 (1984).

[3] Y. Aharonov and J. Anandan, Phys. Rev. Lett. **58**, 1593 (1987).

[4] J. Samuel and R. Bhandari, Phys. Rev. Lett. **60**, 2339 (1988).

[5] *Geometric Phases in Physics*, edited by A. Shapere and F.Wilczek (World Scientific, Singapore, 1989).

[6] S. R. Jain and A. K. Pati, Phys. Rev. Lett. **80**, 650 (1998).

[7] D. Xiao, M. Chang and Q. Niu, Rev. Mod. Phys. **82**, 1959 (2010).

[8] J. A. Jones, V. Vedral, A. Ekert, and G. Castagnoli, Nature **403**, 869 (2000).

[9] A. Uhlmann, Rep. Math. Phys. **24**, 229 (1986).

[10] E. Sjöqvist, A. K. Pati, A. Ekert, J. S. Anandan, M. Ericsson, D. K. L. Oi, and V. Vedral, Phys. Rev. Lett. **85**, 2845 (2000).

[11] K. Singh, D. M. Tong, K. Basu, J. Chen, and J. Du, Phys. Rev. A **67**, 032106 (2003).

[12] D. M. Tong, E. Sjöqvist, L. C. Kwek, and C. H. Oh, Phys. Rev. Lett. **93**, 080405 (2004).

[13] Z. S. Wang, L. C. Lwek, C. H. Lai, and C. H. Oh, Europhys. Lett. **74**, 958 (2006).

[14] J. Du, P. Zou, M. Shi, L. C. Kwek, J. Pan, C. H. Oh, A. Ekert, D. K. L. Oi, and M. Ericsson, Phys. Rev. Lett. **91**, 100403 (2003).

[15] M. Ericsson, D. Achilles, J. T. Barreiro, D. Branning, N. A. Peters, and P. G. Kwiat, Phys. Rev. Lett. **94**, 050401 (2005).

[16] A. Carollo, I. Fuentes-Guridi, M. F. Santos, and V. Vedral, Phys. Rev. Lett. **90**, 160402 (2003); ibid. **92**, 020402 (2004).

[17] A. T. Rezakhani and P. Zanardi, Phys. Rev. A **73**, 052117 (2006).

[18] F. C. Lombardo and P. I. Villar, Phys. Rev. A **74**, 042311 (2006).

[19] J. Chen, J. An, Q. Tong, H. Luo, and C. H. Oh, Phys. Rev. A **81**, 022120 (2010).

[20] W. G. Unruh, Phys. Rev. D **14**, 870 (1976).

[21] W. G. Unruh, Phys. Rev. Lett. **46**, 1351 (1981); L. J. Garay, J. R. Anglin, J. I. Cirac, and P. Zoller, Phys. Rev. Lett. **85**, 4643 (2000); U. Leonhardt, Nature **415**, 406 (2002); T. G. Philbin and et al., Science **319**, 1367 (2008); P. D. Nation, M. P. Blencowe, A. J. Rimberg, and E. Buks, Phys. Rev. Lett. **103**, 087004 (2009).

[22] For a comprehensive review, see for example, L. C. B. Crispino, A. Higuchi, and G. E. A. Matsas, Rev. Mod. Phys. **80**, 787 (2008).

[23] G. E. A. Matsas and D. A. T. Vanzella, Phys. Rev. Lett **87**, 151301 (2001).

[24] A. Higuchi, G. E. A. Matsas and D. Sudarsky, Phys. Rev. D **45**, R3308 (1992); Phys. Rev. D **46**, 3450 (1992).

[25] Danilo T. Alves and Luís C. B. Crispino, Phys. Rev. D **70**, 107703 (2004).

[26] Z. Zhu and H. Yu, Phys. Lett. B **645**, 459(2007); Chin. Phys. Lett. **25**, 1575(2008).

[27] E. Martin-Martinez, I. Fuentes and R. B. Mann, Phys. Rev. Lett. **107**, 131301 (2011).

[28] G. Compagno, R. Passante, and F. Persico, *Atom-Field Interactions and Dressed Atoms* (Cambridge University Press, Cambridge, England, 1995).

[29] F. Benatti and R. Floreanini , Phys. Rev. A **70**, 012112 (2004).

[30] H. Yu and J. Zhang, Phys. Rev. D **77**, 024031 (2007).

[31] H. Yu, Phys. Rev. Lett. **106**, 061101 (2011).

[32] F. M. Cucchietti, J.-F. Zhang, F. C. Lombardo, P. I. Villar, and R. Laflamme, Phys. Rev. Lett. **105**, 240406 (2010).

[33] V. Gorini, A. Kossakowski, and E. C. G. Surdarshan, J. Math. Phys. **17**, 821 (1976); G. Lindblad, Commun. Math. Phys. **48**, 119 (1976).

[34] F. Benatti, R. Floreanini and M. Piani, Phys. Rev. Lett. **91**, 070402 (2003).

[35] R. Passante, Phys. Rev. A **57**, 1950 (1998).

[36] Z. Zhu, H. Yu and S. Lu, Phys. Rev. D **73**, 107501 (2006).

[37] K.-P. Marzlin, S. Ghose, and B. C. Sanders, Phys. Rev. Lett. **93**, 260402 (2004).

[38] J. M. Raimond, M. Brune, and S. Haroche, Rev. Mod. Phys. **73**, 565 (2001).

[39] M. O. Scully, V.V. Kocharovsky, A. Belyanin, E. Fry, and F. Capasso, Phys. Rev. Lett. **91**, 243004 (2003).

[40] E. Sjöqvist, Physics 1, 35 (2008).